STATISTICS QUESTIONS

Step by Step Solutions

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Problem 1: A researcher is interested in the effects of family size on delinquency for a group of offenders and examines families with one to four children. She obtains a sample of 16 families, four of each size, and identifies the number of arrests per child for delinquency. The data is as follows:

	Group 1 4 children n=4	**Group 2** 3 children n=4	**Group 3** 2 children n=4	**Group 4** 1 child n=4
Family 1	10	8	5	4
Family 2	8	8	6	5
Family 3	9	6	7	2
Family 4	10	9	9	2

a) Calculate the total sum of squares.

b) Calculate the mean square (between groups).

c) Calculate the F-ratio

d) Use the Turkey HSD (alpha=0.05) to test for significance between groups. Which groups differed?

e) Based on your results, write a 1-2 paragraph essay that describes your observations obtained from this sample in regard to the effects of family size on delinquency for a group of offenders.

Solution: (a) The following table with descriptive statistics is obtained from the information provided

Obs.	**Group 1**	**Group 2**	**Group 3**	**Group 4**
	10	8	5	4
	8	8	6	5
	9	6	7	2
	10	9	9	2
Mean	**9.25**	**7.75**	**6.75**	**3.25**
St. Dev.	**0.957**	**1.258**	**1.708**	**1.5**

We need to test

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$

$$H_A : \text{Not all the means are equal}$$

With the data found in the table above, we can compute the following values, which are needed to construct the ANOVA table. We have:

$$SS_{Between} = \sum_{i=1}^{k} n_i \left(\overline{x}_i - \overline{\overline{x}}\right)^2$$

from which we get

$$SS_{Between} = 4\left(9.25 - 6.75\right)^2 + 4\left(7.75 - 6.75\right)^2 + 4\left(6.75 - 6.75\right)^2 + 4\left(3.25 - 6.75\right)^2 = 78$$

Now we also see that,

$$SS_{Within} = \sum_{i=1}^{k} \left(n_i - 1\right) s_i^2$$

which implies

$$SS_{Within} = \left(4-1\right)\times 0.957^2 + \left(4-1\right)\times 1.258^2 + \left(4-1\right)\times 1.708^2 + \left(4-1\right)\times 1.5^2 = 23$$

Hence, SS_{Total} = 78+23 = 101

(b) Therefore

$$MS_{Between} = \frac{SS_{Between}}{k-1} = \frac{78}{3} = 26$$

Also, we obtain that

$$MS_{Within} = \frac{SS_{Within}}{N-k-1} = \frac{23}{12} = 1.917$$

(c) Therefore, the F-statistics is computed as

$$F = \frac{MS_{Between}}{MS_{Within}} = \frac{26}{1.917} = 13.5652$$

The critical value for $\alpha = 0.05$, and $df_2 = 12$ is given by

$$F_C = 3.4903$$

and the corresponding p-value is

$$p = \Pr\left(F_{3,12} > 13.5652\right) = 0.000$$

Observed that the p-value is less than the significance level $\alpha = 0.05$, then we reject H_0.

(d) The HSD difference is computed as follows:

$$HSD = Q*\sqrt{\frac{MSE}{n}} = 4.20\sqrt{\frac{1.917}{4}} = 2.91$$

The following table is obtained:

Post hoc analysis

Tukey simultaneous comparison t-values (d.f. = 12)

		Group 4	Group 3	Group 2	Group 1
		3.3	6.8	7.8	9.3
Group 4	3.3				
Group 3	6.8	3.58			
Group 2	7.8	4.60	1.02		
Group 1	9.3	6.13	2.55	1.53	

critical values for experimentwise error rate:

0.05	2.97
0.01	3.89

(e) Based on the above results, we have enough evidence to reject the null hypothesis of equal means, at the 0.05 significance level.

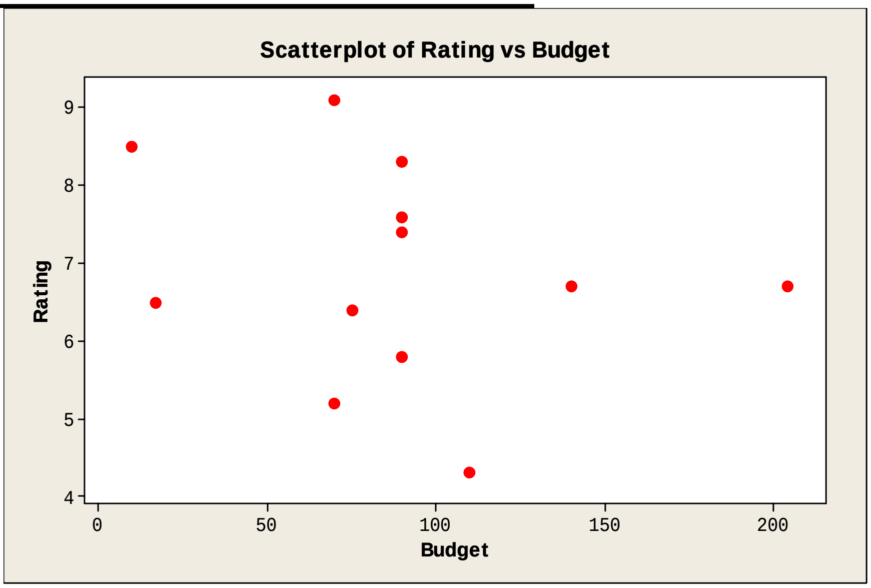
Summarizing, we have the following ANOVA table:

Source	SS	df	MS	F	p-value	Crit. F
Between Groups	78	3	26	13.5652	0.000	3.4903
Within Groups	23	12	1.917			
Total	101	15				

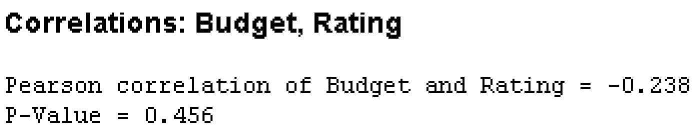
The pairwise differences that are significant are between Group 1 and Group 4, Group 2 and Group, and Group 3 and Group 4. In fact, the mean for Group 4 is significantly lower when compared to the means for groups 1, 2 and 3, respectively.

Problem 2: Movie Success. Using the data in Table 7.2, make a scatter diagram for the relationship between production budget and viewer rating of movies. Estimate the correlation coefficient. Based on these data, do you think a large production budget is likely to result in a movie with a high viewer rating? Explain.

Solution: The scatter plot is shown below.



It seems like there's a mild negative linear relationship between Budget and Rating. The actual correlation coefficient is computed as



Correlations: Budget, Rating

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Pearson correlation of Budget and Rating = -0.238
P-Value = 0.456
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As predicted by the visual trend, the correlation is negative, but since it's very small, the relationship is fairly weak. This means that is not certain that a larger budget will produce a higher rating, as it's not certain that a larger budget will produce a lower rating, but there a inclination to have lower rating with higher budgets.

Problem 3: Which of these models is a better representation of the relationship between students' age and starting salary? Explain your decision.

Solution: As mentioned in the previous part, the model obtained once the outlier was eliminated is relatively similar to the model with n=25 cases, as the regression coefficients don't change dramatically. But still this relatively small difference in coefficients makes a relatively large difference in R^2. In fact, for the model with n = 25 we get R2 = 0.334, and for the model with n = 25 we get R2 = 0.447. This makes the second model (with n = 24) the preferred one. The preferred model is

Starting Salary^ = -67,941.7485 + 3,635.6857* Age

Problem 4:

Compute an imprisonment rate per 1000 population for 2000. Introduce this incarceration rate as an independent variable into the model run in Part B.

Test the hypothesis that the R squared =0.

Does this model fit the data better than the model in Part B above? Explain.

Does each of the independent variables have a statistically significant effect on homicide? Explain.

How strong is the effect of each of the independent variables? Explain.

Which of the independent variables has the stronger effect on the homicide rate? Explain.

Solution: The new variable is computed as

ImprPer1000 = Prison20/pop20

(let us recall that pop20 is already given in 1000's).

The following is obtained with Excel:

Regression Analysis							
	R^2	0.499					
	Adjusted R^2	0.466	n	49			

	R	0.707	k	3			
	Std. Error	1.639	Dep. Var.	**homrt20**			
ANOVA table							
Source	*SS*	*df*	*MS*	*F*	*p-value*		
Regression	120.4481	3	40.1494	14.95	6.87E-07		
Residual	120.8453	45	2.6855				
Total	241.2935	48					
Regression output					*confidence interval*		
variables	*coefficients*	*std. error*	*t (df=45)*	*p-value*	*95% lower*	*95% upper*	*std. coeff.*
Intercept	-1.9451	1.1721	-1.660	.1040	-4.3059	0.4156	0.000
ImprPer1000	0.2430	0.1707	1.423	.1616	-0.1009	0.5869	0.175
sglmom80	24.3703	5.6975	4.277	.0001	12.8949	35.8458	0.539
unempl20	36.9631	27.2997	1.354	.1825	-18.0215	91.9476	0.150

The model is

Homicide Rate in 2000 = -1.9451 + 0.2430* ImprPer1000 + 24.3703* sglmom80 + 36.9631* unempl20

Notice that the model is significant overall, since F(3, 45) = 14.95, p = 0.000000687 < 0.05, so then R^2 is significantly greater than zero.

This model fits only slightly better than the previous one, since now Adj. R^2 = 0.466, which means that in this case the amount of explained variation in the response variable by this model is 46.6%.

Notice that in this model, the variable *sglmom80* is individually significant, with t = 4.277 and p = 0.0001 < 0.05, but the variable *uempl20* is not individually significant, *t = 1.354*, *p = 0.1825 > 0.05*. The variable ImprPer1000 is not significant either, since *t = 1.423, p = 0.1616 > 0.05*.

The effect of *ImprPer1000* and *uempl20* is quite moderate since the standardized coefficients associated to them are less than 0.2 (this is, an increase in one standard deviation in either of the variables brings a change of less than 0.2 standard deviations in the response variable). The variable with the strongest effect is *sglmom80*, with a standardized coefficient of 0.539.

Problem 5: Using the data below, answer the following questions using a table format.

X_i	4	6	3	7
y_i	5	2	-1	4

a. $\sum_{i=1}^{4} x_i$ b. $\sum_{i=1}^{4} y_i$ c. $\sum_{i=1}^{4} x_i y_i$ d. Show that $\sum_{i=1}^{4} x_i \cdot \sum_{i=1}^{4} y_i \neq \sum_{i=1}^{4} x_i y_i$

e. $\sum_{i=1}^{4} x_i^2$ f. $\sum_{i=1}^{4} y_i^2$ g. $\left(\sum_{i=1}^{4} x_i y_i\right)^2$ h. Show that $\sum_{i=1}^{4} x_i^2 \neq \left(\sum_{i=1}^{4} x_i\right)^2$

i. Show that $\sum_{i=1}^{4} (x_i - \overline{x}) = \sum_{i=1}^{4} (y_i - \overline{y}) = 0$

Solution: We have:

	X	**Y**	**X^2**	**Y^2**	**X*Y**
	4	5	16	25	20
	6	2	36	4	12
	3	-1	9	1	-3
	7	4	49	16	28
Sum =	20	10	110	46	57

(a) $\sum_{i=1}^{4} x_i = 20$

(b) $\sum_{i=1}^{4} y_i = 10$

(c) $\sum_{i=1}^{4} x_i y_i = 57$

(d) Notice that $\sum_{i=1}^{4} x_i y_i = 57$, and $\left(\sum_{i=1}^{4} x_i\right)\left(\sum_{i=1}^{4} y_i\right) = 20 \times 10 = 200$, which means that $\sum_{i=1}^{4} x_i y_i \neq \left(\sum_{i=1}^{4} x_i\right)\left(\sum_{i=1}^{4} y_i\right)$ in this case.

(e) $\sum_{i=1}^{4} x_i^2 = 110$

(f) $\sum_{i=1}^{4} y_i^2 = 46$

(g) $\left(\sum_{i=1}^{4} x_i y_i\right)^2 = 57^2 = 3249$

(h) $\left(\sum_{i=1}^{4} x_i\right)^2 = 20^2 = 400$, and $\sum_{i=1}^{4} x_i^2 = 110$, so then $\sum_{i=1}^{4} x_i^2 \neq \sum_{i=1}^{4} x_i$

(i) we get that $\overline{X} = 5, \overline{Y} = 2.5$. Observe that

X	Y	X-Xbar	Y-Ybar
4	5	-1	2.5
6	2	1	-0.5
3	-1	-2	-3.5
7	4	2	1.5
	Sum =	0	0

so then

$$\sum_{i=1}^{4} (x_i - \overline{x}) = \sum_{i=1}^{4} (y_i - \overline{y}) = 0$$